INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, Second Semester, 2010-11
Statistics - IV, Backpaper Examination

1. Suppose we have a random sample $X_1, \dots, X_n$ from a continuous distribution with unknown c.d.f. F. Consider testing $H_0 : F = F_0$, where F_0 is a completely specified c.d.f.
(a) What are the directional and non-directional Kolmogorov-Smirnov test statistics that are useful for this test and when are they used?
(b) Show that the Kolmogorov-Smirnov statistics mentioned in (a) are distribution free under the null hypothesis. [10]

2. Suppose we have a random sample $X_1, \dots, X_n$ from a continuous distribution with c.d.f. F and density f, both of which are completely unknown.
(a) Define the histogram estimate of f.
(b) Show that under appropriate conditions the histogram is a consistent estimator of f. [10]

3. Suppose $X \sim \text{Binomial}(n, \theta)$, where n is fixed but θ is unknown. Consider estimating θ under the loss $L(\theta, a) = \theta^{-2}(\theta - a)^2$.
(a) Show that $\delta_c(X) = cX$ is inadmissible if $c > 1/n$.
(b) Find the Bayes estimator of θ with respect to the prior $\pi(\theta) \propto \theta^{5/2}$, $0 < \theta < 1$. [10]

4. Let X be $N(\theta, 1)$, where $\theta > 0$. Consider the decision problem where the loss function is $L(\theta, a) = (\theta - a)^2$. Consider the two decision rules, $\delta_1(X) = X$ and $\delta_2(X) = X^+ = \max\{X, 0\}$. Show that δ_2 has a uniformly smaller risk than δ_1 for all $\theta > 0$. [10]

5. Consider a two-person, zero-sum game which is strictly determined.
(a) Explain why player I should use the maximin strategy and player II, the minimax strategy, if they are intelligent.
(b) Solve the game with the following loss matrix:

	a_1	a_2	a_3	a_4
θ_1	2	6	0	0
θ_2	0	4	3	1
θ_3	0	3	1	2

[10]